

SAU #19

MATHEMATICS

Updated May 2009

How this is organized

The material in the K-12 Mathematics Curriculum Framework is organized into Grade-Level Expectations (GLEs) and Grade-Span Expectations (GSEs). Both GLEs and GSEs contain statements about what ALL New Hampshire students are expected to know and be able to do. GLEs are standards that are delineated by grade level covering grades K-8. GSEs are standards that cover the high school span. Both the GLEs and GSEs are built upon the extensive work that went into the 1995 K-12 Mathematics Curriculum Framework and the earlier 1993 K-3 monograph.

Both the GLEs and GSEs are organized into four content strands (Number and Operations; Geometry and Measurement; Functions and Algebra; Data, Statistics, and Probability) and two process strands (Problem Solving, Reasoning, and Proof; Communication, Connections, and Representations).

At the beginning of each strand is a purpose statement which places the strand in the context of a K-12 mathematics program. Furthermore, each strand contains a number of big ideas referred to as stems. The stems articulate the main curricular focus and stay the same or similar across grades.

There are two types of standards throughout the document. Those identified for state assessment purposes and those identified for local curriculum and assessment. The state assessment standards appear in bold boxes; all other standards are guidance for local curriculum and assessment.

The organization of the GLEs and GSEs is further clarified in *About the Mathematics GLEs*, *Reading the Mathematics GLEs*, *About the Mathematics GSEs*, and *Reading the Mathematics GSEs*.

GOALS FOR MATHEMATICS EDUCATION

- All students will develop a firm grounding in number sense that includes computational fluency.
- All students will develop a basic understanding of key concepts and principles central to the study of geometry, algebra, probability, and data analysis, while appreciating the interrelationships of all areas of mathematics.
- All students will develop strong mathematical problem solving and reasoning abilities.
- All students will develop positive attitudes about mathematics.
- All students will develop the ability to use appropriate technology to solve mathematical problems.
- All students will develop the ability to communicate their understanding of mathematics effectively.

THE LEARNING ENVIRONMENT

Knowledge needs to be well connected. Students who have memorized facts and procedures without understanding often do not know when or how to apply them. When students learn mathematics by doing mathematics, by exploring and discussing concepts in the context of physical situations, what emerges from these experiences are skills that are anchored in understanding and clarity. The students not only know the basic procedures, but also know how to apply them to new situations.

- students are actively involved in doing mathematics;
- problem solving, thinking, reasoning, and communicating are everyday activities;
- central mathematical concepts are understood;
- an appropriate balance between application and acquisition of knowledge and skills exists;
- manipulatives are used, when appropriate, to connect conceptual to procedural understanding;
- technology is used in appropriate ways;
- the curriculum is coherent and well-articulated across grade levels; and
- assessment is an integral part of instruction.

About the Mathematics GLEs

This set of New Hampshire Grade-Level Expectations (GLEs) for K-8 includes expectations that will be assessed on the state-level assessment and others that will be a local curriculum and assessment option (i.e., guidance for local curriculum and assessment but not assessed on the state assessment). Those GLEs that will be assessed on the state assessment are boxed with bold outlines. Furthermore, these assessment GLEs are common across NH, RI, and VT and will be assessed through the New England Common Assessment Program (NECAP). The assessment GLEs in this document can be interpreted as describing expectations for the end of the grade identified or the beginning of the next grade. The local content GLEs were developed in collaboration with the Rhode Island Department of Education. As you review the New Hampshire GLEs in Mathematics the following are important to understand.

- 1) The GLEs are organized into four content strands (Number and Operations; Functions and Algebra; Data, Statistics, and Probability; Geometry and Measurement) and two process strands (Problem-Solving, Reasoning, and Proof; Communications, Representations, and Connections).
- 2) Since it is crucial that process standards (problem-solving, reasoning, proof, communication, connections, and representations) are not seen separate from content standards, the process standards have been imbedded throughout the content strands (e.g., M:F&A:4:1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables or sequences; and writes a rule in words or^{sc} symbols to find the next case.). This mirrors classroom instruction as in most classes, as students are learning content knowledge, instruction is also focusing on improving their abilities in problem solving, reasoning, and communication; furthermore, students are looking for and making appropriate connections, and they are able to understand and use multiple representations of mathematical ideas. Since it is crucial that students are strong in both content and process knowledge, we have included two local process strands following the content strands. These process strands are in addition to the process standards that are embedded in the content standards and are included to help guide local curriculum, assessment, and instruction. The process standards have been separated by grade-spans (K–2, 3–5, 6–8, and 9–12). The K–2, 3–5, and 6–8 spans have been included in this section of the document. Each span should be thought of as building upon the skills and concepts in the previous span.
- 3) Each GLE includes a **bolded** statement called the “stem.” Each “stem” is the same or similar across the grades for a given GLE, and is meant to communicate the main curricular and instructional focus of the GLE across the grades.
- 4) The unbolded text within a GLE indicates the expectations for that given grade-level.
- 5) The underlined text indicates the concepts and skills that are new to a given grade. (Note: Sometimes nothing is underlined within a GLE. In these situations, examine other GLEs across the strand to identify the differences or check the Depth of Knowledge table in the test specifications.
- 6) Each GLE is coded for the content strand, grade level, and the GLE “stem” number (e.g., M:F&A:7:3: The “M” stands for mathematics, the “F&A” stands for the Functions and Algebra strand, the “7” indicates a standard for grade 7 to be assessed in the fall of grade 8, and the “3” stands for stem 3 or the 3rd big idea in the Functions and Algebra strand). See the diagram on page 9.
- 7) Unless otherwise specified the number parameters defined in the Number and Operations strand for a particular grade-level apply to all GLEs at that grade-level.
- 8) All the concepts and skills identified at a given grade level that are boxed with bold outlines are “fair game” for state assessment purposes. However, conjunctions in this document have specific meaning. The conjunction “and” separates parts of a GLE that will be assessed every year (to the extent possible), while the conjunction “or” separates parts of the GLE that may be assessed each year, but will be more likely to be assessed over several years. In some situations “or^{sc}” is used. While students will have choices on strategies they use or methods to communicate their thinking throughout the assessment, there are special cases that the NECAP states thought it was necessary to communicate to the test developer that students should not be required to use a specific method (e.g., “...writes in words or^{sc} symbols...”).

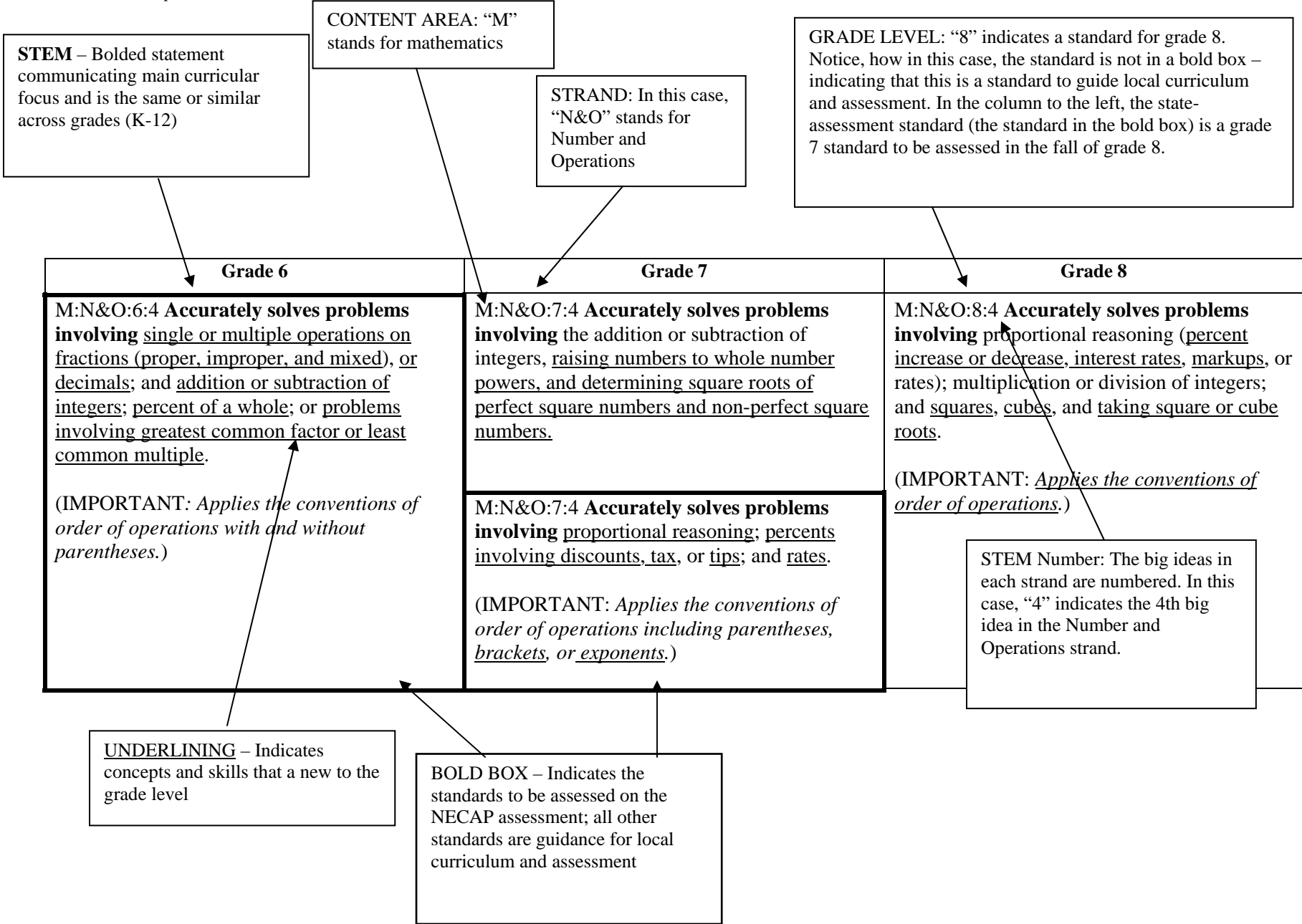


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Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:K:1	Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 12 through investigations that apply the concepts of equivalency in composing or decomposing numbers using models, explanations, or other representations; and positive fractional numbers ($\frac{1}{2}$) as “fair share” (i.e., equal sized parts or sets) using models, explanations, or other representations.
M:N&O:K:2	Demonstrates understanding of the relative magnitude of numbers from 0 to 20 through investigations that demonstrate one-to-one correspondence; that compare whole numbers to each other or to benchmark whole numbers (5, 10); that demonstrate an understanding of the relation of inequality when comparing whole numbers by using “1 more” or “1 less”; that connect numbers orally and written as numerals to the quantities that they represent using models, representations, or number lines.

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:K:3	Demonstrates conceptual understanding of mathematical operations through investigations involving addition and subtraction of whole numbers (from 0 to 10) by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)
M:N&O:K:4	No standard at this grade
M:N&O:K:5	Demonstrates understanding of monetary value through investigation involving knowing the names and values for coins (penny, nickel and dime).
M:N&O:K:6	Mentally adds and subtracts whole numbers by naming the number that is one more or one less than the original number. (IMPORTANT: <i>The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)
M:N&O:K:7	Makes estimates of the number of objects in a set (up to 20) by making and revising estimates as objects are counted (e.g., A student estimates the number of pennies in a jar as 20. Then the student counts the first 10 and makes another estimate based on those that have been counted and those that remain in the jar). (IMPORTANT: <i>Estimation should be imbedded instructionally throughout all strands.</i>)

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:K:1	Uses properties, attributes, composition, or decomposition to sort or classify polygons (triangles, squares, rectangles, rhombi, trapezoids, and hexagons) or objects by using one non-measurable or measurable attribute; and recognizes, names, and builds polygons and circles in the environment.
M:G&M:K:2	No standard at this grade
M:G&M:K:3	No standard at this grade
M:G&M:K:4	No standard at this grade
M:G&M:K:5	No standard at this grade
M:G&M:K:6	No standard at this grade

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:K:7	Demonstrates conceptual understanding of measurable attributes using comparative language to describe and compare attributes of objects (length [longer, shorter], height [taller, shorter], weight [heavier, lighter], temperature [warmer, cooler], and capacity [more, less]); and compares objects visually and with direct comparison.
M:G&M:K:8	Determines elapsed and accrued time as it relates to calendar patterns (days of the week, yesterday, today, and tomorrow), the sequence of events in a day; and identifies a clock and calendar as measurement tools (days of week, months of the year).
M:G&M:K:9	Demonstrates understanding of spatial relationships using location and position by using positional words to locate and describe where an object is found in the environment.

Mathematics: *Functions and Algebra*
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:K:1	I Identifies and extends to specific cases a variety of patterns (sequences of shapes, sounds, movement, colors, and letters) by extending the pattern to the next one, two or three elements, or by translating AB patterns across formats (e.g., an abb pattern can be represented as snap, clap, clap or red, yellow, yellow) or by identifying number patterns in the environment.
M:F&A:K:2	No standard at this grade
M:F&A:K:3	No standard at this grade
M:F&A:K:4	No standard at this grade

Mathematics: *Data, Statistics, and Probability*
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:K:1	<p>Interprets a given representation created by the class (models and tally charts) to answer questions related to the data, or to analyze the data to formulate conclusions using words, diagrams, or verbal/scribed responses to express answers.</p> <p><i>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:K:2)</i></p>
M:DSP:K:2	Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal (e.g., Have there been more, less, or the same number of cloudy days compared to sunny days this week?).
M:DSP:K:3	No standard at this grade
M:DSP:K:4	No standard at this grade
M:DSP:K:5	No standard at this grade
M:DSP:K:6	No standard at this grade

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills ~ K-2
M:PRP:2:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Formulate and solve multi-step problems from everyday and mathematical situations. • Solve problems using a variety of strategies (e.g., working backwards, looking for patterns and relationships; guess and check; making tables, charts, or organized lists; solving a simpler version of a problem, drawing a diagram; or creating a model) • Verify and interpret results with respect to the original problem. • Determine if the solution of a problem is reasonable. • Solve problems using manipulative, graphs, charts, diagrams, and calculators. • Demonstrate that a problem may be solved in more than one way. • Exhibit confidence in their ability to solve problems independently and in groups. • Display increasing perseverance, and persistence in problem solving.
M:PRP:2:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Use models, known facts, properties, and relationships to explain their thinking. • Justify solution processes and answers (e.g., "I chose this method to solve the problem because..."). • Draw conclusions using inductive reasoning. • Identify the missing information needed to find a solution to a given story problem. • Use patterns and relationships to analyze mathematical situations (e.g., count by fives).

Mathematics: Communication, Connections, and Representations
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ K-2
M:CCR:2:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Demonstrate mathematical communication through discussion, reading, writing, listening, and responding, individually and in groups. • Discuss relationships between everyday language and mathematical language and symbols (e.g., words that mean something different in mathematics and in everyday life). • Explain conclusions, thought processes, and strategies in problem-solving situations. • Discuss, illustrate, and write about mathematical concepts and relationships. • Draw pictures and use objects to illustrate mathematical concepts.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills ~ Grades K-2
M:CCR:2:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Create and use age level appropriate representations to organize, record, and communicate mathematical ideas (e.g., students should recognize the relationship among seven counters, seven tally marks, and the symbol 7). • Select, apply, and translate among mathematical representations to solve problems (e.g., representing fractions with circles, with geoboards, and with pattern blocks). • Link different representations. • Use representations to model and interpret physical, social, and mathematical phenomena. • Use conventional and self-generated (invented) representations and connect them. • Realize that any representation is subject to multiple interpretations (e.g., drawings and graphs can be read in a different way).
M:CCR:2:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • Link conceptual and procedural knowledge (e.g., they will know that when they “regroup,” they are simply changing the representation of the minuend, but not its value). • Recognize and use mathematics in other curriculum areas (e.g., science, social studies). • Recognize and use mathematics in their daily lives (e.g., graphs, tables, or maps). • Identify mathematical situations occurring in literature for children. • Identify examples of geometry in nature, art, and architecture.

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:1:1	<p>Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 100 using <u>place value</u>, by applying the concepts of equivalency in composing or decomposing numbers; and <u>in expanded notation</u> using models, explanations, or other representations; and positive fractional numbers (benchmark fractions: $\frac{a}{2}$, $\frac{a}{3}$, or $\frac{a}{4}$, where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in <u>area models where the denominator is equal to the number of parts in the whole</u> using models, explanations, or other representations.</p>

Grade - 1		Mathematics
Mathematics: <i>Number and Operations</i> continued Grade Level Expectations		
Content Strand Grade Stem Number	Concepts and Skills	
M:N&O:1:2	Demonstrates understanding of the relative magnitude of numbers <u>from 0 to 100</u> by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (5, 10, <u>25</u> , <u>50</u> , <u>75</u> , <u>100</u>); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using “1 more”, “1 less”, “ <u>5 more</u> ”, “ <u>5 less</u> ”, “ <u>10 more</u> ”, “ <u>10 less</u> ”; and by connecting <u>number words</u> (from 0 to 20) and numerals (from 0 to 100) to the quantities and <u>positions</u> that they represent using investigations, models, representations, or number lines.	
M:N&O:1:3	Demonstrates conceptual understanding of mathematical operations involving addition and subtraction of whole numbers (<u>from 0 to 30</u>) by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)	
M:N&O:1:4	No standard at this grade	
M:N&O:1:5	Demonstrates understanding of monetary value by knowing the names and values for coins (penny, nickel, dime, and <u>quarter</u>); and <u>by adding collections of like coins together to a sum no greater than \$1.00.</u>	
M:N&O:1:6	Mentally adds and subtracts whole numbers by naming the number that is one or <u>two</u> more or less than the original number; and adds and subtracts whole number facts through ten (addends whose sum is at most 10 and related subtraction facts). (IMPORTANT: <i>The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)	

Grade - 1		Mathematics
Mathematics: <i>Number and Operations</i> continued Grade Level Expectations		
Content Strand Grade Stem Number	Concepts and Skills	
M:N&O:1:7	<p>Makes estimates of the number of objects in a set (<u>up to 30</u>) and revises estimates as objects are counted (e.g., A student estimates the number of pennies in a jar as 28. Then the student counts the first 10 and makes another estimate based on those that have been counted and those that remain in the jar.).</p> <p>(IMPORTANT: <i>Estimation should be imbedded instructionally throughout all strands.</i>)</p>	
M:N&O:1:8	<p>Applies properties of numbers (odd, even, composition, and decomposition [e.g., 5 is the same as $2 + 3$]) and field properties (commutative and identity for addition) to solve problems and to simplify computations involving whole numbers.</p>	

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:1:1	Uses properties, attributes, composition, or decomposition to sort or classify polygons (triangles, squares, rectangles, rhombi, trapezoids, and hexagons) or objects by <u>a combination of two</u> non-measurable or measurable attributes; and recognizes, names, builds, and <u>draws</u> polygons and circles in the environment.
M:G&M:1:2	No standard at this grade
M:G&M:1:3	Given an example of a three-dimensional geometric shape (rectangular prisms, cylinders, or spheres) finds examples of objects in the environment that are of the same geometric shape (e.g., show a wooden cylinder and students identify common objects of the same shape).
M:G&M:1:4	Demonstrates conceptual understanding of congruency by making mirror images and creating shapes that have line symmetry.
M:G&M:1:5	No standard at this grade

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:1:6	Demonstrates conceptual understanding of the length/height of a two-dimensional object using non-standard units (e.g. comparing objects to trains of small cubes, using iterations of a small unit to measure an object).
M:G&M:1:7	Demonstrates conceptual understanding of measurable attributes using comparative language to describe and compare attributes of objects (length [longer, shorter], height [taller, shorter], weight [heavier, lighter], temperature [warmer, cooler], and capacity [more, less]); compares objects visually, with direct comparison, and <u>using non-standard units</u> .
M:G&M:1:8	Determines elapsed and accrued time as it relates to calendar patterns (days of the week, <u>months of the year</u>), the sequence of events in a day; and <u>recognizes an hour and "on the ½ hour"</u> .
M:G&M:1:9	Demonstrates understanding of spatial relationships using location and position by using positional words (e.g., close by, on the right, underneath, above, beyond) to describe one location in reference to another <u>on a map, in a diagram</u> , and in the environment.

Mathematics: *Functions and Algebra*
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:1:1	Identifies and extends to specific cases a variety of patterns (repeating and <u>growing [numeric and non-numeric]</u>) represented in models, tables, or sequences by extending the pattern to the next one, two, or <u>three elements</u> , by <u>finding a missing element</u> (e.g., 2, 4, 6, <u> </u> , 10), or by translating repeating patterns across formats (e.g., an abb pattern can be represented as snap, clap, clap; or red, yellow, yellow; or 1, 2, 2).
M:F&A:1:2	No standard at this grade
M:F&A:1:3	No standard at this grade
M:F&A:1:4	Demonstrates conceptual understanding of equality by <u>finding the value that will make an open sentence true</u> (e.g., <u>$2 + \square = 7$</u>) using models, verbal explanations, or written equations. <u>(limited to one operation and limited to use addition or subtraction)</u>

Mathematics: *Data, Statistics, and Probability*
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:1:1	<p>Interprets a given representation created by the class (models, tally charts, <u>pictographs with one-to-one correspondence</u>, and <u>tables</u>) to answer questions related to the data, or to analyze the data to formulate conclusions using words, diagrams, or verbal/scribed responses to express answers.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:1:2.</i>)</p>
M:DSP:1:2	Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal.
M:DSP:1:3	No standard at this grade
M:DSP:1:4	No standard at this grade
M:DSP:1:5	For a probability event in which the sample space may or may not contain equally likely outcomes, groups use experiments to describe the likelihood or chance of an event (using “more likely,” “less likely”, or “equally likely”).
M:DSP:1:6	No standard at this grade

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills ~ K-2
M:PRP:2:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Formulate and solve multi-step problems from everyday and mathematical situations. • Solve problems using a variety of strategies (e.g., working backwards, looking for patterns and relationships; guess and check; making tables, charts, or organized lists; solving a simpler version of a problem, drawing a diagram; or creating a model) • Verify and interpret results with respect to the original problem. • Determine if the solution of a problem is reasonable. • Solve problems using manipulative, graphs, charts, diagrams, and calculators. • Demonstrate that a problem may be solved in more than one way. • Exhibit confidence in their ability to solve problems independently and in groups. • Display increasing perseverance, and persistence in problem solving.
M:PRP:2:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Use models, known facts, properties, and relationships to explain their thinking. • Justify solution processes and answers (e.g., "I chose this method to solve the problem because..."). • Draw conclusions using inductive reasoning. • Identify the missing information needed to find a solution to a given story problem. • Use patterns and relationships to analyze mathematical situations (e.g., count by fives).

Mathematics: Communication, Connections, and Representations
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ K-2
M:CCR:2:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Demonstrate mathematical communication through discussion, reading, writing, listening, and responding, individually and in groups. • Discuss relationships between everyday language and mathematical language and symbols (e.g., words that mean something different in mathematics and in everyday life). • Explain conclusions, thought processes, and strategies in problem-solving situations. • Discuss, illustrate, and write about mathematical concepts and relationships. • Draw pictures and use objects to illustrate mathematical concepts.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills ~ Grades K-2
M:CCR:2:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Create and use age level appropriate representations to organize, record, and communicate mathematical ideas (e.g., students should recognize the relationship among seven counters, seven tally marks, and the symbol 7). • Select, apply, and translate among mathematical representations to solve problems (e.g., representing fractions with circles, with geoboards, and with pattern blocks). • Link different representations. • Use representations to model and interpret physical, social, and mathematical phenomena. • Use conventional and self-generated (invented) representations and connect them. • Realize that any representation is subject to multiple interpretations (e.g., drawings and graphs can be read in a different way).
M:CCR:2:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • Link conceptual and procedural knowledge (e.g., they will know that when they “regroup,” they are simply changing the representation of the minuend, but not its value). • Recognize and use mathematics in other curriculum areas (e.g., science, social studies). • Recognize and use mathematics in their daily lives (e.g., graphs, tables, or maps). • Identify mathematical situations occurring in literature for children. • Identify examples of geometry in nature, art, and architecture.

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:2:1	<div style="border: 2px solid black; padding: 10px;"> <p>Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 199 using place value, by applying the concepts of equivalency in composing or decomposing numbers (e.g., $34 = 17 + 17$; $34 = 29 + 5$); and in expanded notation (e.g., $141 = 1 \text{ hundred} + 4 \text{ tens} + 1 \text{ one}$ or $141 = 100 + 40 + 1$) using models, explanations, or other representations; and positive fractional numbers (benchmark fractions: $\frac{a}{2}$, $\frac{a}{3}$, or $\frac{a}{4}$, where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the denominator is equal to the number of parts in the whole using models, explanations, or other representations</p> </div>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:2:2	<p>Demonstrates understanding of the relative magnitude of numbers from <u>0 to 199</u> by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (10, 25, 50, 75, 100, <u>125</u>, <u>150</u>, or <u>175</u>); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using “1 more”, “1 less”, “10 more”, “10 less”, “<u>100 more</u>”, or “<u>100 less</u>”; or by connecting number words and numerals to the quantities they represent using models, number lines, or explanations.</p>
M:N&O:2:3	<p>Demonstrates conceptual understanding of mathematical operations involving addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)</p>
M:N&O:2:4	No standard at this grade
M:N&O:2:5	<p>Demonstrates understanding of monetary value by <u>adding coins together to a value no greater than \$1.99 and representing the result in dollar notation; making change from \$1.00 or less, or recognizing equivalent coin representations of the same value (values up to \$1.99).</u></p>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:2:6	<p>Mentally adds and subtracts whole number facts <u>through 20</u> (addends whose sum is <u>at most 20 and related subtraction facts</u>); names the number that is <u>10 more or less</u> than the original number, and mentally adds and subtracts two-digit multiples of ten (e.g., $60 + 80$, $90 - 30$).</p> <p><i>(IMPORTANT: The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)</i></p>
M:N&O:2:7	<p>Makes estimates of the number of objects in a set (<u>up to 50</u>) by selecting an appropriate method of estimation.</p> <p><i>(IMPORTANT: The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.)</i></p>
M:N&O:2:8	<p>Applies properties of numbers (odd and even) and field properties (commutative for addition, identity for addition, and <u>associative for addition</u>) to solve problems and to simplify computations involving whole numbers.</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:2:1	Uses properties, attributes, composition, or decomposition to sort or classify polygons or objects by <u>a combination of two or more</u> non-measurable or measurable attributes.
M:G&M:2:2	No standard at this grade
M:G&M:2:3	No standard at this grade
M:G&M:2:4	Demonstrates conceptual understanding of congruency by <u>composing and decomposing two-dimensional objects using models or explanations (e.g., using triangular pattern blocks to construct a figure congruent to the hexagonal pattern block)</u> ; and <u>uses line symmetry to demonstrate congruent parts within a shape</u> .
M:G&M:2:5	No standard at this grade

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:2:6	Demonstrates conceptual understanding of perimeter and area by using models or manipulatives to surround and cover polygons.
M:G&M:2:7	Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. Benchmarks in Appendix B.
M:G&M:2:8	No standard at this grade
M:G&M:2:9	Demonstrates understanding of spatial relationships using location and position by using positional language <u>in two- and three- dimensional situations to describe and interpret relative positions (e.g., above the surface of the desk, below the triangle on the paper); and creates and interprets simple maps and names locations on simple coordinate grids.</u>

Mathematics: *Functions and Algebra*
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:2:1	<div style="border: 2px solid black; padding: 5px;"> Identifies and extends to specific cases a variety of patterns (linear and non-numeric) represented in models, tables, or sequences by extending the pattern to the next element, or finding a missing element (e.g., 2, 4, 6, ____, 10). </div>
M:F&A:2:2	No standard at this grade
M:F&A:2:3	No standard at this grade
M:F&A:2:3	<div style="border: 2px solid black; padding: 5px;"> Demonstrates conceptual understanding of equality by finding the value that will make an open sentence true (e.g., $2 + \square = 7$). (limited to one operation and limited to use addition or subtraction) </div>

Grade - 2	Mathematics
Mathematics: <i>Data, Statistics, and Probability</i> Grade Level Expectations	

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:2:1	<div style="border: 2px solid black; padding: 10px;"> <p>Interprets a given representation (pictographs with one-to-one correspondence, <u>line plots</u>, tally charts, or tables) to answer questions related to the data, or to analyze the data to formulate conclusions.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:2:2.</i>)</p> </div>
M:DSP:2:2	<div style="border: 2px solid black; padding: 10px;"> <p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal.</p> </div>
M:DSP:2:3	No standard at this grade
M:DSP:2:4	<div style="border: 2px solid black; padding: 10px;"> <p>Uses counting techniques to solve problems involving combinations using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or^{sc} others); (e.g., How many ways can you make 50 cents using nickels, dimes, and quarters?)</p> </div>
M:DSP:2:5	For a probability event in which the sample space may or may not contain equally likely outcomes, uses experiments to describe the likelihood or chance of an event using “more likely,” “less likely,” “equally likely,” <u>certain</u> or <u>impossible</u> .

Grade - 2		Mathematics
Mathematics: <i>Data, Statistics, and Probability</i> continued Grade Level Expectations		
Content Strand Grade Stem Number	Concepts and Skills	
M:DSP:2:6	<p><u>In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions.</u></p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:2:2</i>)</p>	

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills ~ K-2
M:PRP:2:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Formulate and solve multi-step problems from everyday and mathematical situations. • Solve problems using a variety of strategies (e.g., working backwards, looking for patterns and relationships; guess and check; making tables, charts, or organized lists; solving a simpler version of a problem, drawing a diagram; or creating a model) • Verify and interpret results with respect to the original problem. • Determine if the solution of a problem is reasonable. • Solve problems using manipulative, graphs, charts, diagrams, and calculators. • Demonstrate that a problem may be solved in more than one way. • Exhibit confidence in their ability to solve problems independently and in groups. • Display increasing perseverance, and persistence in problem solving.
M:PRP:2:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Use models, known facts, properties, and relationships to explain their thinking. • Justify solution processes and answers (e.g., "I chose this method to solve the problem because..."). • Draw conclusions using inductive reasoning. • Identify the missing information needed to find a solution to a given story problem. • Use patterns and relationships to analyze mathematical situations (e.g., count by fives).

Mathematics: Communication, Connections, and Representations
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ K-2
M:CCR:2:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Demonstrate mathematical communication through discussion, reading, writing, listening, and responding, individually and in groups. • Discuss relationships between everyday language and mathematical language and symbols (e.g., words that mean something different in mathematics and in everyday life). • Explain conclusions, thought processes, and strategies in problem-solving situations. • Discuss, illustrate, and write about mathematical concepts and relationships. • Draw pictures and use objects to illustrate mathematical concepts.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills ~ Grades K-2
M:CCR:2:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Create and use age level appropriate representations to organize, record, and communicate mathematical ideas (e.g., students should recognize the relationship among seven counters, seven tally marks, and the symbol 7). • Select, apply, and translate among mathematical representations to solve problems (e.g., representing fractions with circles, with geoboards, and with pattern blocks). • Link different representations. • Use representations to model and interpret physical, social, and mathematical phenomena. • Use conventional and self-generated (invented) representations and connect them. • Realize that any representation is subject to multiple interpretations (e.g., drawings and graphs can be read in a different way).
M:CCR:2:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • Link conceptual and procedural knowledge (e.g., they will know that when they “regroup,” they are simply changing the representation of the minuend, but not its value). • Recognize and use mathematics in other curriculum areas (e.g., science, social studies). • Recognize and use mathematics in their daily lives (e.g., graphs, tables, or maps). • Identify mathematical situations occurring in literature for children. • Identify examples of geometry in nature, art, and architecture.

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:3:1	<div style="border: 2px solid black; padding: 10px;"> <p>Demonstrates conceptual understanding of rational numbers with respect to: whole numbers <u>from 0 to 999</u> through equivalency, composition, decomposition, or place value using models, explanations, or other representations; and positive fractional numbers (benchmark fractions: $\frac{a}{2}$, $\frac{a}{3}$, $\frac{a}{4}$, $\frac{a}{6}$, or $\frac{a}{8}$, where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the number of parts in the whole is equal to the denominator; and <u>decimals (within a context of money)</u> as a part of 100 using models, explanations, or other representations</p> </div>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:3:2	<p>Demonstrates understanding of the relative magnitude of numbers <u>from 0 to 999</u> by ordering whole numbers; by comparing whole numbers to benchmark whole numbers (100, 250, 500, 750); or by <u>comparing whole numbers to each other</u>; and <u>comparing or identifying equivalent positive fractional numbers</u> ($a/2$, $a/3$, $a/4$, where a is a whole number greater than 0 and less than or equal to the denominator) using models, number lines, or explanations.</p>
M:N&O:3:3	<p>Demonstrates conceptual understanding of mathematical operations <u>by describing or illustrating the inverse relationship between addition and subtraction of whole numbers</u>; and <u>the relationship between repeated addition and multiplication using models, number lines, or explanations</u>.</p>
M:N&O:3:4	<p>Accurately solves problems involving addition and subtraction with regrouping; the concept of multiplication; and addition or subtraction of decimals (in the context of money).</p>
M:N&O:3:5	No standard at this grade
M:N&O:3:6	<p>Mentally adds and subtracts whole number facts through 20 (addends whose sum is at most 20 and related subtraction facts); <u>adds two-digit and one-digit whole numbers</u>; <u>adds combinations of two-digit and three-digit whole numbers that are multiples of ten</u> (e.g., $60 + 50$, $300 + 400$, $320 + 90$); <u>subtracts a one-digit whole number from a two-digit whole number</u> (e.g., $37 - 5$); and <u>subtracts two-digit whole numbers that are multiples of ten and three-digit whole numbers that are multiples one hundred</u> (e.g., $50 - 20$, $500 - 200$).</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:3:7	<p><u>Makes estimates in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, and evaluating the reasonableness of solutions appropriate to grade level GLEs across content strands.</u></p> <p><i>(IMPORTANT: The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.)</i></p>
M:N&O:3:8	<p>Applies properties of numbers (odd, even, and <u>multiplicative property of zero for single-digit whole numbers</u> [$6 \times 0 = 0$]) and field properties (commutative for addition, associative for addition, <u>identity for multiplication</u>, and <u>commutative for multiplication for single-digit whole numbers</u> [e.g., $3 \times 4 = 4 \times 3$]) to solve problems and to simplify computations involving whole numbers.</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:3:1	<div style="border: 2px solid black; padding: 5px;"> <p><u>Uses properties or attributes of angles (number of angles) or sides (number of sides or length of sides) or composition or decomposition of shapes to identify, describe, or distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or circles.</u></p> </div>
M:G&M:3:2	No standard at this grade
M:G&M:3:3	No standard at this grade
M:G&M:3:4	<p>Demonstrates conceptual understanding of congruency by <u>matching congruent figures using reflections, translations, and rotations (flips, slides, and turns) (e.g., recognizing when pentominoes are reflections, translations and rotations of each other); composing and decomposing two- and three-dimensional objects using models or explanations (e.g., Given a cube, students use blocks to construct a congruent cube.); and by using line symmetry to demonstrate congruent parts within a shape.</u></p>
M:G&M:3:5	Demonstrates conceptual understanding of similarity by identifying similar shapes.

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:3:6	<p>Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles <u>on grids</u> using a variety of models or manipulatives. <u>Expresses all measures using appropriate units.</u></p>
M:G&M:3:7	<p>Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.</p> <p>Benchmarks in Appendix B.</p>
M:G&M:3:8	No standard at this grade
M:G&M:3:9	<p>Demonstrates understanding of spatial relationships using location and position <u>by interpreting and giving directions from one location to another (e.g., classroom to the gym, from school to home) using positional words; and between locations on a map or coordinate grid (first quadrant) using positional words or compass directions.</u></p>
M:G&M:3:10	<p>Demonstrates conceptual understanding of spatial reasoning and visualization by copying, comparing, and drawing models of triangles, squares, rectangles, rhombi, trapezoids, hexagons, and circles; and builds models of rectangular prisms from three-dimensional representations.</p>

Mathematics: Functions and Algebra
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:3:1	<div style="border: 2px solid black; padding: 5px;"> Identifies and extends to specific cases a variety of patterns (linear and non-numeric) represented in models, tables, or sequences by extending the pattern to the next one, <u>two</u>, or <u>three elements</u>, or finding missing <u>elements</u>. </div>
M:F&A:3:2	No standard at this grade
M:F&A:3:3	No standard at this grade
M:F&A:3:4	<div style="border: 2px solid black; padding: 5px;"> Demonstrates conceptual understanding of equality by showing equivalence between two expressions using models or different representations of the expressions; or by finding the value that will make an open sentence true (e.g., $2 + \square = 7$). (limited to one operation and limited to use addition, subtraction, or <u>multiplication</u>) </div>

Mathematics: Data, Statistics, and Probability
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:3:1	<div style="border: 2px solid black; padding: 5px;"> <p>Interprets a given representation (line plots, tally charts, tables, or <u>bar graphs</u>) to answer questions related to the data, to analyze the data to formulate conclusions, or to <u>make predictions</u>.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:3:2.</i>)</p> </div>
M:DSP:3:2	<div style="border: 2px solid black; padding: 5px;"> <p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using <u>most frequent (mode)</u>, <u>least frequent</u>, <u>largest</u>, or <u>smallest</u>.</p> </div>
M:DSP:3:3	<p>Organizes and displays data using tables, tally charts, and bar graphs, to answer questions related to the data, to analyze the data to formulate conclusions, to make predictions, or to solve problems.</p>
M:DSP:3:3	<div style="border: 2px solid black; padding: 5px;"> <p>Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in M:DSP:3:1.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:3:2.</i>)</p> </div>

Grade - 3		Mathematics
Mathematics: <i>Data, Statistics, and Probability</i> continued Grade Level Expectations		
Content Strand Grade Stem Number	Concepts and Skills	
M:DSP:3:4	Uses counting techniques to solve problems involving combinations and <u>simple permutations</u> using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or ^{sc} others).	
M:DSP:3:5	For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event using “more likely,” “less likely,” “equally likely,” certain, or impossible and tests the prediction through experiments; and determines if a game is fair.	
M:DSP:3:5	For a probability event in which the sample space may or may not contain equally likely outcomes, determines the likelihood of the occurrence of an event (using “more likely”, “less likely”, or “equally likely”).	
M:DSP:3:6	<p>In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:3:2.</i>)</p>	

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills - 3-5
M:PRP:5:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Determine the reasonableness of solutions to real-world problems. • Generalize solutions and apply strategies to new problem situations. • Add to the repertoire of problem-solving strategies (e.g., looking for similar problems) and use those strategies in more sophisticated ways. • Solve problems with multiple solutions, recognize when a problem has no solution, and recognize problems where more information is needed. • Translate results of a computation into solutions that fit the real-world problem (e.g., when a computation shows that one needs 3.2 gallons of paint to paint a room, how much paint do you buy?).
M:PRP:5:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Draw conclusions and solve problems using elementary deductive reasoning and reasoning by analogy. • Make and defend conjectures and generalizations. • Use models, known facts, properties, and relationships to explain thinking and to justify answers and solution processes. • Recognize the pervasive use and power of reasoning as a part of mathematics.

Mathematics: Communication, Connections, and Representations
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ 3-5
M:CCR:5:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Discuss mathematical ideas and write convincing arguments. • Understand, explain, analyze, and evaluate mathematical arguments and conclusions presented by others. • Ask clarifying and extending questions related to mathematics they have heard or read about. • Understand and appreciate the economy and power of mathematical symbolism and its role in the development of mathematics. • Demonstrate an understanding of mathematical concepts and relationships through a variety of methods (e.g., writing, graphing, charts, diagrams, number sentences, or symbols). • Use a variety of technologies (e.g., computers, calculators, video, probes) to represent and communicate mathematical ideas.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills - Grades 3-5
M:CCR:5:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Use physical models and diagrams to represent important mathematical ideas (e.g., multiplication). • Use appropriate representations to solve problems or to portray, clarify, or extend a mathematical idea. • Recognize equivalent representations of concepts and procedures and translate among them as appropriate (for example, understand how the addition of whole numbers, fractions, and decimals are related).
M:CCR:5:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • See mathematics as an integrated whole. • Recognize relationships among different topics in mathematics. • Recognize and use mathematics in other curriculum areas and in their daily lives. • Link concepts and procedures. • Use mathematical skills, concepts, and applications in other disciplines (e.g., graphs in social studies, patterns in art, or music and geometry in technology education).

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:4:1	<div style="border: 2px solid black; padding: 10px;"> <p>Demonstrates conceptual understanding of rational numbers with respect to:</p> <p>whole numbers <u>from 0 to 999,999</u> through <u>equivalency, composition, decomposition, or place value using models, explanations, or other representations</u>; and <u>positive fractional numbers</u> (benchmark fractions: $\frac{a}{2}$, $\frac{a}{3}$, $\frac{a}{4}$, $\frac{a}{5}$, $\frac{a}{6}$, $\frac{a}{8}$, or $\frac{a}{10}$, where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area, set, or <u>linear models</u> where the number of parts in the whole are equal to, and a <u>multiple or factor of the denominator</u>; and <u>decimals as hundredths</u> within the context of money, or tenths <u>within the context of metric measurements (e.g., 2.3 cm)</u> using models, explanations, or other representations.</p> </div>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:4:2	<p>Demonstrates understanding of the relative magnitude of numbers from 0 to 999,999 by ordering or comparing whole numbers; and ordering, comparing, or identifying equivalent proper positive <u>fractional numbers</u>; <u>or decimals</u> using models, number lines, or explanation</p>
M:N&O:4:3	<p>Demonstrates conceptual understanding of mathematical operations by describing or illustrating <u>the relationship between repeated subtraction and division (no remainders)</u>; <u>the inverse relationship between multiplication and division of whole numbers</u>; or <u>the addition or subtraction of positive fractional numbers with like denominators</u> using models, number lines, or explanations.</p>
M:N&O:4:4	<p>Accurately solves problems involving multiple operations on whole numbers or the use of <u>the properties of factors and multiples</u>; and addition or subtraction of <u>decimals and positive proper fractions with like denominators</u>. (Multiplication limited to 2 digits by 2 digits, and division limited to 1 digit divisors.)</p> <p>(IMPORTANT: <u>Applies the conventions of order of operations where the left to right computations are modified only by the use of parentheses.</u>)</p>
M:N&O:4:5	No standard at this grade

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:4:6	<p>Mentally adds and subtracts whole number facts through 20 (addends whose sum is at most 20 and related subtraction facts); <u>multiplies whole number facts to a product of 100</u>, and <u>calculates related division facts</u>; adds <u>two-digit whole numbers</u>, combinations of two-digit and 3-digit whole numbers that are multiples of ten, and <u>4-digit whole numbers that are multiples of 100</u> (limited to two addends) (e.g., $67 + 24$; $320 + 430$; $320 + 90$; $1,300 + 1,400$); and subtracts a one-digit whole number from a two-digit whole number (e.g., $67 - 9$); and <u>subtracts combinations of two-digit and three-digit whole numbers that are multiples of ten</u> (e.g., $50 - 20$, $230 - 80$, $520 - 200$).</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:4:7	<p>Makes estimates in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:4:8	<p>Applies properties of numbers (odd, even, <u>multiplicative property of zero</u>, and <u>remainders</u>) and field properties (<u>commutative</u>, <u>associative</u>, and identity) to solve problems and to simplify computations.</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:4:1	<div style="border: 2px solid black; padding: 5px;"> <p>Uses properties or attributes of angles (number of angles) or sides (number of sides, length of sides, <u>parallelism</u>, or <u>perpendicularity</u>) to identify, describe, or distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or <u>octagons</u>; or <u>classify angles relative to 90°</u> as more than, less than, or equal to.</p> </div>
M:G&M:4:2	No standard at this grade
M:G&M:4:3	<div style="border: 2px solid black; padding: 5px;"> <p>Uses properties or attributes (shape of bases or number of lateral faces) to identify, compare, or describe three-dimensional shapes (rectangular prisms, triangular prisms, <u>cylinders</u>, or <u>spheres</u>).</p> </div>
M:G&M:4.4	<div style="border: 2px solid black; padding: 5px;"> <p>Demonstrates conceptual understanding of congruency by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), or as the result of composing or decomposing shapes using models or explanations.</p> </div>

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:4:5	<p>Demonstrates conceptual understanding of similarity by applying scales on maps, or applying characteristics of similar figures (same shape but not necessarily the same size) to identify similar figures, or to solve problems involving similar figures. Describes relationships using models or^{sc} explanations.</p>
M:G&M:4:6	<p>Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles, <u>polygons or irregular shapes on grids</u> using a variety of models, manipulatives, or <u>formulas</u>. Expresses all measures using appropriate units.</p>
M:G&M:4:7	<p>Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.</p> <p>Benchmarks in Appendix B.</p>
M:G&M:4:8	No standard at this grade
M:G&M:4:9	Demonstrates understanding of spatial relationships using location and position by interpreting and giving directions between locations on a map or coordinate grid (first quadrant); <u>plotting points in the first quadrant in context (e.g., games, mapping)</u> ; and finding <u>the horizontal and vertical distances between points on a coordinate grid in the first quadrant</u> .
M:G&M:4:10	Demonstrates conceptual understanding of spatial reasoning and visualization by copying, comparing, and drawing models of triangles, squares, rectangles, rhombi, trapezoids, hexagons, <u>octagons</u> , and circles; and builds models of rectangular prisms from <u>two-</u> or three-dimensional representations.

Mathematics: Functions and Algebra
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:4:1	<div style="border: 2px solid black; padding: 5px;"> <p>Identifies and <u>extends to specific cases</u> a variety of patterns (linear and <u>nonlinear</u>) represented in models, tables or sequences; and <u>writes a rule in words or^{sc} symbols to find the next case.</u></p> </div>
M:F&A:4:2	<p>Demonstrates conceptual understanding of linear relationships ($y = kx$) as a constant rate of change by identifying, describing, or comparing situations that represent constant rates of change.</p>
M:F&A:4:3	<div style="border: 2px solid black; padding: 5px;"> <p>Demonstrates conceptual understanding of algebraic expressions by using letters or symbols to represent unknown quantities to write simple linear algebraic expressions involving any one of the four operations; or by evaluating simple linear algebraic expressions using whole numbers.</p> </div>
M:F&A:4:4	<div style="border: 2px solid black; padding: 5px;"> <p>Demonstrates conceptual understanding of equality by showing equivalence between two expressions using models or different representations of the expressions, by <u>simplifying numerical expressions where left to right computations may be modified only by the use of parentheses [e.g., $14 - (2 \times 5)$]</u> (expressions consistent with the parameters of M:F&A:4-3), and by <u>solving one-step linear equations of the form $ax = c$, $x \pm b = c$, where a, b, and c are whole numbers with $a \neq 0$.</u></p> </div>

Mathematics: Data, Statistics, and Probability
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:4:1	<p>Interprets a given representation (line plots, tables, bar graphs, <u>pictographs</u>, or <u>circle graphs</u>) to answer questions related to the data, to analyze the data to formulate or <u>justify</u> conclusions, to make predictions, or to <u>solve problems</u>.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:4:2.)</p>
M:DSP:4:2	<p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using <u>measures of central tendency</u> (median or mode), or <u>range</u>.</p>
M:DSP:4:3	<p>Organizes and displays data using tables, <u>line plots</u>, bar graphs, and <u>pictographs</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:4:2.)</p>
M:DSP:4:4	<p>Uses counting techniques to solve problems in context involving combinations or simple permutations (e.g., Given a map – Determine the number of paths from point A to point B.) using a variety of strategies (e.g., organized lists, tables, tree diagrams, or^{sc} others</p>

Mathematics: *Data, Statistics, and Probability* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:4:5	For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event <u>as a part to whole relationship</u> (e.g., two out of five, zero out of five, five out of five) and tests the prediction through experiments; and determines if a game is fair.
M:DSP:4:5	For a probability event in which the sample space may or may not contain equally likely outcomes, determines the <u>theoretical probability of an event and expresses the result as part to whole (e.g., two out of five)</u> .
M:DSP:4:6	<p>In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions; and <u>asks new questions and makes connections to real world situations</u>.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:4:2</i>)</p>

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills - 3-5
M:PRP:5:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Determine the reasonableness of solutions to real-world problems. • Generalize solutions and apply strategies to new problem situations. • Add to the repertoire of problem-solving strategies (e.g., looking for similar problems) and use those strategies in more sophisticated ways. • Solve problems with multiple solutions, recognize when a problem has no solution, and recognize problems where more information is needed. • Translate results of a computation into solutions that fit the real-world problem (e.g., when a computation shows that one needs 3.2 gallons of paint to paint a room, how much paint do you buy?).
M:PRP:5:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Draw conclusions and solve problems using elementary deductive reasoning and reasoning by analogy. • Make and defend conjectures and generalizations. • Use models, known facts, properties, and relationships to explain thinking and to justify answers and solution processes. • Recognize the pervasive use and power of reasoning as a part of mathematics.

Mathematics: Communication, Connections, and Representations
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ 3-5
M:CCR:5:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Discuss mathematical ideas and write convincing arguments. • Understand, explain, analyze, and evaluate mathematical arguments and conclusions presented by others. • Ask clarifying and extending questions related to mathematics they have heard or read about. • Understand and appreciate the economy and power of mathematical symbolism and its role in the development of mathematics. • Demonstrate an understanding of mathematical concepts and relationships through a variety of methods (e.g., writing, graphing, charts, diagrams, number sentences, or symbols). • Use a variety of technologies (e.g., computers, calculators, video, probes) to represent and communicate mathematical ideas.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills - Grades 3-5
M:CCR:5:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Use physical models and diagrams to represent important mathematical ideas (e.g., multiplication). • Use appropriate representations to solve problems or to portray, clarify, or extend a mathematical idea. • Recognize equivalent representations of concepts and procedures and translate among them as appropriate (for example, understand how the addition of whole numbers, fractions, and decimals are related).
M:CCR:5:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • See mathematics as an integrated whole. • Recognize relationships among different topics in mathematics. • Recognize and use mathematics in other curriculum areas and in their daily lives. • Link concepts and procedures. • Use mathematical skills, concepts, and applications in other disciplines (e.g., graphs in social studies, patterns in art, or music and geometry in technology education).

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:5:1	<div style="border: 2px solid black; padding: 10px;"> <p>Demonstrates conceptual understanding of rational numbers with respect to:</p> <p>whole numbers from <u>0 to 9,999,999</u> through equivalency, composition, decomposition, or place value using models, explanations, or other representations; and positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, <u>twelfths</u>, fifths, or <u>powers of ten (10, 100, 1000)</u>), decimals (<u>to thousandths</u>), or <u>benchmark percents (10%, 25%, 50%, 75% or 100%)</u> as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.</p> </div>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:5:2	<p>Demonstrates understanding of the relative magnitude of numbers by ordering, comparing, or identifying equivalent positive <u>fractional numbers</u>, decimals, or <u>benchmark percents within number formats (fractions to fractions, decimals to decimals, or percents to percents)</u>; or <u>integers</u> in context using models or number lines.</p>
M:N&O:5:3	<p>Demonstrates conceptual understanding of mathematical operations by <u>adding and subtracting decimals and positive proper fractions with unlike denominators</u>.</p>
M:N&O:5:3	<p>Demonstrates conceptual understanding of mathematical operations by describing or illustrating the <u>meaning of a remainder with respect to division of whole numbers</u> using models, explanations, or <u>solving problems</u>.</p>
M:N&O:5:4	<p>Accurately solves problems involving multiple operations on whole numbers or the use of the properties of factors, multiples, <u>prime</u>, or <u>composite numbers</u>; and addition or subtraction of <u>fractions (proper) and decimals to the hundredths place</u>. (<u>Division of whole numbers by up to a two-digit divisor</u>.)</p> <p>(IMPORTANT: <i>Applies the conventions of order of operations <u>with and without parentheses</u>.</i>)</p>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:5:5	No standard at this grade
M:N&O:5:6	<p><u>Mentally calculates change back from \$1.00, \$5.00, and \$10.00; calculates multiplication and related division facts to a product of 144; multiplies a two-digit whole number by a one-digit whole number (e.g., 45×5), two-digit whole numbers that are multiples of ten (e.g., 50×60), a three-digit whole number that is a multiple of 100 by a two- or three-digit number which is a multiple of 10 or 100, respectively (e.g., 400×50, 400×600); and divides 3- and 4-digit multiples of powers of ten by their compatible factors (e.g., $360 \div 6$; $360 \div 60$; $3600 \div 6$; $3600 \div 60$; $3600 \div 600$; $360 \div 12$; $360 \div 120$; $3600 \div 12$; $3600 \div 120$; $3600 \div 1200$).</u></p> <p>(IMPORTANT: <i>The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:5:7	<p>Makes estimates in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, <u>determining the level of accuracy needed given the situation, analyzing the effect of the estimation method on the accuracy of results</u>, and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:5:8	<p>Applies properties of numbers (odd, even, and <u>divisibility</u>) and field properties (commutative, associative, identity, and <u>distributive</u>) to solve problems and to simplify computations.</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:5:1	<div style="border: 2px solid black; padding: 5px;"> <p>Uses properties or attributes of angles (<u>right</u>, <u>acute</u>, or <u>obtuse</u>) or sides (<u>number of congruent sides</u>, parallelism, or perpendicularity) to identify, describe, <u>classify</u>, or distinguish among <u>different types of triangles</u> (<u>right</u>, <u>acute</u>, <u>obtuse</u>, <u>equiangular</u>, or <u>equilateral</u>) or <u>quadrilaterals</u> (<u>rectangles</u>, <u>squares</u>, <u>rhombi</u>, <u>trapezoids</u>, or <u>parallelograms</u>).</p> </div>
M:G&M:5:2	No standard at this grade
M:G&M:5:3	<div style="border: 2px solid black; padding: 5px;"> <p>Uses properties or attributes (shape of bases, number of lateral faces, or <u>number of bases</u>) to identify, compare, or describe three-dimensional shapes (<u>rectangular prisms</u>, <u>triangular prisms</u>, <u>cylinders</u>, <u>spheres</u>, <u>pyramids</u>, or <u>cones</u>).</p> </div>
M:G&M:5:4	No standard at this grade

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:5:5	Demonstrates conceptual understanding of <u>similarity</u> by describing the proportional effect on the linear dimensions of triangles and rectangles when scaling up or down while preserving angle <u>measures</u> , or by solving related problems (including applying scales on maps). Describes effects using models or ^{sc} explanations.
M:G&M:5:6	<p>Demonstrates conceptual understanding of perimeter of polygons, and the area of rectangles or <u>right triangles</u> through models, manipulatives, or formulas, the area of polygons or <u>irregular figures</u> on grids, and <u>volume of rectangular prisms (cubes)</u> using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.</p>
M:G&M:5:7	<p>Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.</p> <p>Benchmarks in Appendix B.</p>
M:G&M:5:8	No standard at this grade
M:G&M:5:9	Demonstrates understanding of spatial relationships using location and position by interpreting and giving directions between locations on a map or coordinate grid (<u>all four quadrants</u>); <u>plotting points in four quadrants in context (e.g., games, mapping, identifying the vertices of polygons as they are reflected, rotated, and translated)</u> ; and determining horizontal and vertical distances between points on a coordinate grid in the first quadrant.
M:G&M:5:10	Demonstrates conceptual understanding of spatial reasoning and visualization by building models of rectangular and <u>triangular prisms</u> , <u>cones</u> , <u>cylinders</u> , and <u>pyramids</u> from two- or three-dimensional representations.

Mathematics: Functions and Algebra
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:5:1	<div style="border: 2px solid black; padding: 5px;"> Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, or <u>in problem situations</u>; and writes a rule in words or^{sc} symbols <u>for finding specific cases of a linear relationship</u>. </div>
M:F&A:5:2	Demonstrates conceptual understanding of linear relationships ($y = kx$) as a constant rate of change by identifying, describing, or comparing situations that represent constant rates of change (e.g., tell a story given a line graph about a trip).
M:F&A:5:3	<div style="border: 2px solid black; padding: 5px;"> Demonstrates conceptual understanding of algebraic expressions by using letters to represent unknown quantities to write <u>linear algebraic expressions</u> involving <u>any two</u> of the four operations; or by evaluating <u>linear algebraic expressions</u> using whole numbers. </div>
M:F&A:5:4	<div style="border: 2px solid black; padding: 5px;"> Demonstrates conceptual understanding of equality by showing equivalence between two expressions using models or different representations of the expressions (expressions consistent with the parameters of <u>M:F&A:5-3</u>), by solving one-step linear equations of the form $ax = c$, $x \pm b = c$, or $x/a = c$, where a, b, and c are whole numbers with $a \neq 0$; or by <u>determining which values of a replacement set make the equation (multi-step of the form $ax \pm b = c$ where a, b, and c are whole numbers with $a \neq 0$) a true statement</u> (e.g., $2x + 3 = 11$, $\{x: x = 2, 3, 4, 5\}$). </div>

Mathematics: Data, Statistics, and Probability
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:5:1	<p>Interprets a given representation (tables, bar graphs, circle graphs, or <u>line graphs</u>) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p><i>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:5:2.)</i></p>
M:DSP:5:2	<p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (<u>mean</u>, median, or mode) or range to <u>analyze situations</u>, or to <u>solve problems</u>.</p>
M:DSP:5:3	<p>Organizes and displays data using tables, bar graphs, or <u>line graphs</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p>
M:DSP:5:3	<p>Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in <u>M:DSP:5:1</u>.</p> <p><i>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:5:2.)</i></p>

Mathematics: *Data, Statistics, and Probability* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:5:4	No standard at this grade
M:DSP:5:5	For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event as a <u>fraction</u> and tests the prediction through experiments; and determines if a game is fair.
M:DSP:5:5	For a probability event in which the sample space may or may not contain equally likely outcomes, determines the <u>experimental</u> or theoretical probability of an event and expresses the result as a fraction.
M:DSP:5:6	<p>In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions; and asks new questions and makes connections to real world situations.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M;DSP:5:2.</i>)</p>

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills - 3-5
M:PRP:5:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Determine the reasonableness of solutions to real-world problems. • Generalize solutions and apply strategies to new problem situations. • Add to the repertoire of problem-solving strategies (e.g., looking for similar problems) and use those strategies in more sophisticated ways. • Solve problems with multiple solutions, recognize when a problem has no solution, and recognize problems where more information is needed. • Translate results of a computation into solutions that fit the real-world problem (e.g., when a computation shows that one needs 3.2 gallons of paint to paint a room, how much paint do you buy?).
M:PRP:5:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Draw conclusions and solve problems using elementary deductive reasoning and reasoning by analogy. • Make and defend conjectures and generalizations. • Use models, known facts, properties, and relationships to explain thinking and to justify answers and solution processes. • Recognize the pervasive use and power of reasoning as a part of mathematics.

Mathematics: Communication, Connections, and Representations
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ 3-5
M:CCR:5:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Discuss mathematical ideas and write convincing arguments. • Understand, explain, analyze, and evaluate mathematical arguments and conclusions presented by others. • Ask clarifying and extending questions related to mathematics they have heard or read about. • Understand and appreciate the economy and power of mathematical symbolism and its role in the development of mathematics. • Demonstrate an understanding of mathematical concepts and relationships through a variety of methods (e.g., writing, graphing, charts, diagrams, number sentences, or symbols). • Use a variety of technologies (e.g., computers, calculators, video, probes) to represent and communicate mathematical ideas.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills - Grades 3-5
M:CCR:5:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Use physical models and diagrams to represent important mathematical ideas (e.g., multiplication). • Use appropriate representations to solve problems or to portray, clarify, or extend a mathematical idea. • Recognize equivalent representations of concepts and procedures and translate among them as appropriate (for example, understand how the addition of whole numbers, fractions, and decimals are related).
M:CCR:5:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • See mathematics as an integrated whole. • Recognize relationships among different topics in mathematics. • Recognize and use mathematics in other curriculum areas and in their daily lives. • Link concepts and procedures. • Use mathematical skills, concepts, and applications in other disciplines (e.g., graphs in social studies, patterns in art, or music and geometry in technology education).

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:6:1	<div style="border: 2px solid black; padding: 5px;"> Demonstrates conceptual understanding of rational numbers with respect to <u>ratios</u> (comparison of two whole numbers by division a/b, $a : b$, and $a \div b$, where $b \neq 0$); and <u>rates</u> (e.g., a out of b, 25%) using models, explanations, or other representations. </div>
M:N&O:6:2	<div style="border: 2px solid black; padding: 5px;"> Demonstrates understanding of the relative magnitude of numbers by ordering or comparing <u>numbers with whole number bases and whole number exponents</u> (e.g., 3^3, 4^3), integers, or <u>rational numbers within and across number formats</u> (fractions, decimals, or whole number percents from 1–100) using number lines or <u>equality and inequality symbols</u>. </div>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:6:3	Demonstrates conceptual understanding of mathematical operations by adding and subtracting <u>positive fractions and integers</u> ; and <u>multiplying and dividing fractions and decimals</u> .
M:N&O:6:3	Demonstrates conceptual understanding of mathematical operations by <u>describing or illustrating the meaning of a power by representing the relationship between the base (whole number) and the exponent (whole number) (e.g., 3^3, 4^3)</u> ; and <u>the effect on the magnitude of a whole number when multiplying or dividing it by a whole number, decimal, or fraction</u> .
M:N&O:6:4	Accurately solves problems involving <u>single or multiple operations on fractions (proper, improper, and mixed), or decimals</u> ; and <u>addition or subtraction of integers; percent of a whole; or problems involving greatest common factor or least common multiple</u> . (IMPORTANT: <i>Applies the conventions of order of operations with and without parentheses.</i>)
M:N&O:6:5	No standard at this grade

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:6:6	<p>Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates change back from \$5.00, \$10.00, <u>\$20.00</u>, <u>\$50.00</u>, and <u>\$100.00</u>; multiplies a two-digit whole number by a one-digit number whole number (e.g., 45×5), two-digit whole numbers that are multiples of ten (e.g., 50×60), a three-digit whole number that is a multiple of 100 by a two- or three-digit number which is a multiple of 10 or 100, respectively (e.g., 400×50, 400×600); divides 3- and 4-digit multiples of powers of ten by their compatible factors (e.g., $360 \div 6$; $360 \div 60$; $3600 \div 6$; $3600 \div 60$; $3600 \div 600$; $360 \div 12$; $360 \div 120$; $3600 \div 12$; $3600 \div 120$; $3600 \div 1200$); and <u>determines the part of a whole number using benchmark percents (1%, 10%, 25%, 50%, and 75%)</u>.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:6:7	<p>Makes estimates in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, determining the level of accuracy needed given the situation, analyzing the effect of the estimation method on the accuracy of results, and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs across content strands</u>.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:6:8	<p>Applies properties of numbers (odd, even, <u>positive</u>, <u>negative</u>, remainders, divisibility, and <u>prime factorization</u>) and field properties (commutative, associative, identity [<u>including the multiplicative property of one</u>, e.g., $1 = 2/2$ and $2/2 \times 3/4 = 6/8$, so $3/4 = 6/8$], distributive, and <u>additive inverses</u>) to solve problems and to simplify computations.</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:6:1	<div style="border: 2px solid black; padding: 10px;"> <p>Uses properties or attributes of angles (right, acute, or obtuse) or sides (number of congruent sides, parallelism, or perpendicularity) to identify, describe, classify, or distinguish among different types of triangles (right, acute, obtuse, equiangular, <u>scalene</u>, <u>isosceles</u>, or equilateral) or quadrilaterals (rectangles, squares, rhombi, trapezoids, or parallelograms).</p> </div>
M:G&M:6:2	No standard at this grade
M:G&M:6:3	<div style="border: 2px solid black; padding: 10px;"> <p>Uses properties or attributes (shape of bases, number of lateral faces, number of bases, <u>number of edges</u>, or <u>number of vertices</u>) to identify, compare, or describe three-dimensional shapes (rectangular prisms, triangular prisms, cylinders, spheres, pyramids, or cones).</p> </div>

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:6:4	Demonstrates conceptual understanding of congruency by <u>predicting and describing the transformational steps (reflections, translations, and rotations) needed to show congruence (including the degree of rotation) and as the result of composing and decomposing two- and three-dimensional objects using models or explanations</u> ; and using line and rotational symmetry to demonstrate congruent parts within a shape.
M:G&M:6:5	Demonstrates conceptual understanding of similarity by describing the proportional effect on the linear dimensions of <u>polygons or circles</u> when scaling up or down while preserving the angles of polygons, or by solving related problems (including applying scales on maps). Describes effects using models or ^{sc} explanations.
M:G&M:6:6	Demonstrates conceptual understanding of perimeter of polygons, the area of <u>quadrilaterals or triangles</u> , and <u>the volume of rectangular prisms</u> by using models, formulas, or by <u>solving problems</u> ; and <u>demonstrates understanding of the relationships of circle measures (radius to diameter and diameter to circumference) by solving related problems</u> . Expresses all measures using appropriate units.
M:G&M:6:7	Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. Benchmarks in Appendix B.
M:G&M:6:8	No standard at this grade
M:G&M:6:9	No standard at this grade
M:G&M:6:10	No standard at this grade

Mathematics: Functions and Algebra
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:6:1	<div style="border: 2px solid black; padding: 10px;"> <p>Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, <u>graphs</u>, or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or <u>writes a rule in words or^{sc} symbols for finding specific cases of a nonlinear relationship</u>; and <u>writes an expression or^{sc} equation using words or^{sc} symbols to express the generalization of a linear relationship (e.g., twice the term number plus 1 or^{sc} $2n + 1$)</u>.</p> </div>
M:F&A:6:2	<div style="border: 2px solid black; padding: 10px;"> <p>Demonstrates conceptual understanding of linear relationships ($y = kx$, $y = mx + b$) as a constant rate of change by <u>constructing or interpreting graphs of real occurrences and describing the slope of linear relationships (faster, slower, greater, or smaller) in a variety of problem situations</u>; and <u>describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change</u>.</p> </div>

Mathematics: *Functions and Algebra* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:6:3	<p>Demonstrates conceptual understanding of algebraic expressions by using letters to represent unknown quantities to write linear algebraic expressions involving <u>any of the four operations and consistent with order of operations expected at this grade level</u>; or by evaluating linear algebraic expressions (<u>including those with more than one variable</u>); or by <u>evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 3x - 2$)</u>.</p>
M:F&A:6:4	<p>Demonstrates conceptual understanding of equality by showing equivalence between two expressions using models or different representations of the expressions (expressions consistent with the parameters of <u>M:F&A:6-3</u>), <u>solving multi-step linear equations of the form $ax \pm b = c$, where a, b, and c are whole numbers with $a \neq 0$</u>.</p>

Mathematics: *Data, Statistics, and Probability*
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:6:1	<p>Interprets a given representation (circle graphs, line graphs, or <u>stem-and-leaf plots</u>) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:6:2.</i>)</p>
M:DSP:6:2	<p>Analyzes patterns, trends or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode) or <u>dispersion (range)</u> to analyze situations, or to solve problems.</p>
M:DSP:6:3	<p>Organizes and displays data using tables, line graphs, or <u>stem-and-leaf plots</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:6:2.</i>)</p>
M:DSP:6:4	<p>Uses counting techniques to solve problems in context involving combinations or simple permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, <u>Fundamental Counting Principle</u>, or^{sc} others).</p>

Mathematics: *Data, Statistics, and Probability* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:6:5	For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the theoretical probability of an event and tests the prediction through experiments and <u>simulations</u> ; and <u>designs fair games</u> .
M:DSP:6:5	For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an <u>event in a problem-solving situation</u> .
M:DSP:6:6	<p>In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions; and asks new questions and makes connections to real world situations.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:6:2.</i>)</p>

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills - 6-8
M:PRP:8:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Use problem-solving strategies appropriately and effectively for a given situation. • Determine, collect and organize the relevant information needed to solve real-world problems. • Apply integrated problem-solving strategies to solve problems in the physical, natural, and social sciences and in pure mathematics. • Use technology when appropriate to solve problems. • Reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.
M:PRP:8:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Draw logical conclusions and make generalizations using deductive and inductive reasoning. • Formulate, test, and justify mathematical conjectures and arguments. • Construct and determine the validity of a mathematical argument or a solution. • Apply mathematical reasoning skills in other disciplines

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ 6-8
M:CCR:8:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.
M:CCR:8:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Use models and technology to develop equivalent representations of the same mathematical concept. • Use and create representations to solve problems and organize their thoughts and ideas. • Convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).

Grade Span - 6 – 8

Mathematics

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade
Stem Number

Concepts and Skills - Grades 6-8

M:CCR:8:3

Students will recognize, explore, and develop mathematical connections and be able to:

- Connect new mathematical ideas to those already studied and build upon them.
- Understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon).
- Explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.
- Explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:7:1	Demonstrates conceptual understanding of rational numbers with respect to square roots of perfect squares, rates, and proportional reasoning.
M:N&O:7:1	<div style="border: 2px solid black; padding: 5px;"> <p>Demonstrates conceptual understanding of rational numbers with respect to <u>percents as a means of comparing the same or different parts of the whole when the wholes vary in magnitude (e.g., 8 girls in a classroom of 16 students compared to 8 girls in a classroom of 20 students, or 20% of 400 compared to 50% of 100); and percents as a way of expressing multiples of a number (e.g., 200% of 50) using models, explanations, or other representations.</u></p> </div>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:7:2	<p>Demonstrates understanding of the relative magnitude of numbers by ordering, comparing, or identifying equivalent rational numbers <u>across number formats</u>, numbers with whole number bases and whole number exponents (e.g., 3^3, 4^3), integers, <u>absolute values</u>, or <u>numbers represented in scientific notation</u> using number lines or equality and inequality symbols.</p>
M:N&O:7:3	<p>Demonstrates conceptual understanding of operations with <u>integers and whole number exponents</u> (where the base is a whole number) using models, diagrams, or explanations.</p>
M:N&O:7:4	<p>Accurately solves problems involving the addition or subtraction of integers, <u>raising numbers to whole number powers</u>, and determining square roots of perfect square numbers and non-perfect square numbers.</p>
M:N&O:7:4	<p>Accurately solves problems involving <u>proportional reasoning</u>; <u>percents involving discounts, tax, or tips</u>; and <u>rates</u>.</p> <p>(IMPORTANT: <i>Applies the conventions of order of operations including parentheses, <u>brackets</u>, or <u>exponents</u>.</i>)</p>
M:N&O:7:5	No standard at this grade

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:7:6	<p>Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates <u>benchmark perfect squares and related square roots</u> (e.g., 1^2, 2^2, ..., 12^2, 15^2, 20^2, 25^2, 100^2, 1000^2); determines the <u>part of a number</u> using benchmark percents and <u>related fractions</u> (1%, 10%, 25%, $33\frac{1}{3}\%$, 50%, $66\frac{2}{3}\%$, 75%, and 100%) (e.g., 25% of 16; $33\frac{1}{3}\%$ of 330).</p> <p>(IMPORTANT: <i>The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:7:7	<p>Makes estimates in a given situation (<u>including tips, discounts, and tax</u>) by identifying when estimation is appropriate, selecting the appropriate method of estimation, determining the level of accuracy needed given the situation, analyzing the effect of the estimation method on the accuracy of results, and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>
M:N&O:7:8	<p>Applies properties of numbers (odd, even, positive, negative, remainders, divisibility, and prime factorization) and field properties (commutative, associative, identity, distributive, <u>inverses</u>) to solve problems and to simplify computations, and <u>demonstrates conceptual understanding of field properties as they apply to subsets of the real numbers</u> (e.g., <u>the set of whole numbers does not have additive inverses, the set of integers does not have multiplicative inverses</u>).</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:7:1	<div style="border: 2px solid black; padding: 10px;"> <p>Uses <u>properties of angle relationships</u> resulting from two or three intersecting lines (<u>adjacent angles, vertical angles, straight angles, or angle relationships formed by two non-parallel lines cut by a transversal</u>), or <u>two parallel lines cut by a transversal to solve problems</u>.</p> </div>
M:G&M:7:2	<div style="border: 2px solid black; padding: 10px;"> <p>Applies theorems or relationships (triangle inequality or sum of the measures of interior angles of regular polygons) to solve problems.</p> </div>
M:G&M:7:3	No standard at this grade

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:7:4	<u>Applies the concepts of congruency by solving problems on a coordinate plane involving reflections, translations, or rotations.</u>
M:G&M:7:5	<u>Applies concepts of similarity by solving problems involving scaling up or down and their impact on angle measures, linear dimensions and areas of polygons, and circles when the linear dimensions are multiplied by a constant factor. Describes effects using models or^{sc} explanations.</u>
M:G&M:7:6	<u>Demonstrates conceptual understanding of the area of circles or the area or perimeter of composite figures (quadrilaterals, triangles, or parts of circles), and the surface area of rectangular prisms, or volume of rectangular prisms, triangular prisms, or cylinders using models, formulas, or by solving related problems. Expresses all measures using appropriate units.</u>
M:G&M:7:7	No standard at this grade
M:G&M:7:8	No standard at this grade
M:G&M:7:9	No standard at this grade
M:G&M:7:10	<u>Demonstrates conceptual understanding of spatial reasoning and visualization by sketching three-dimensional solids; and draws nets of rectangular and triangular prisms, cylinders, and pyramids and uses the nets as a technique for finding surface area.</u>

Mathematics: *Functions and Algebra*
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:7:1	<p>Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; and <u>generalizes a linear relationship using words and symbols; generalizes a linear relationship to find a specific case; or writes an expression or^{sc} equation using words or^{sc} symbols to express the generalization of a nonlinear relationship.</u></p>
M:F&A:7:2	<p>Demonstrates conceptual understanding of linear relationships ($y = kx$; $y = mx + b$) as a constant rate of change by <u>solving problems involving the relationship between slope and rate of change, by describing the meaning of slope in concrete situations, or informally determining the slope of a line from a table or graph; and distinguishes between constant and varying rates of change in concrete situations represented in tables or graphs;</u> or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.</p>

Mathematics: *Functions and Algebra* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:7:3	<p>Demonstrates conceptual understanding of algebraic expressions by using letters to represent unknown quantities to write <u>algebraic expressions (including those with whole number exponents or more than one variable)</u>; or by evaluating <u>algebraic expressions (including those with whole number exponents or more than one variable)</u>; or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 5x^3 - 2$).</p>
M:F&A:7:4	<p>Demonstrates conceptual understanding of equality by showing equivalence between two expressions (<u>expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level</u>) using models or different representations of the expressions, solving multi-step linear equations of the form $ax \pm b = c$ with $a \neq 0$, $ax \pm b = cx \pm d$ with $a, c \neq 0$, and $(x/a) \pm b = c$ with $a \neq 0$, where a, b, c and d are whole numbers; or by <u>translating a problem-solving situation into an equation consistent with the parameters of the type of equations being solved for this grade level</u>.</p>

Mathematics: Data, Statistics, and Probability
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:7:1	<p style="border: 2px solid black; padding: 5px;">Interprets a given representation (circle graphs, <u>scatter plots that represent discrete linear relationships</u>, or <u>histograms</u>) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:7:2.</i>)</p>
M:DSP:7:2	<p style="border: 2px solid black; padding: 5px;">Analyzes patterns, trends, or distributions in data in a variety of contexts <u>by solving problems</u> using measures of central tendency (mean, median, or mode), dispersion (range or variation), or <u>outliers</u> to analyze situations to <u>determine their effect on mean, median, or mode</u>; and <u>evaluates the sample from which the statistics were developed (bias)</u>.</p>
M:DSP:7:3	<p>Organizes and displays data using tables, line graphs, <u>scatter plots</u>, and <u>circle graphs</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p>
M:DSP:7:3	<p style="border: 2px solid black; padding: 5px;">Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in <u>M:DSP:7:1</u>.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:6:2.</i>)</p>

Mathematics: *Data, Statistics, and Probability* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:7:4	Uses counting techniques to solve problems in context involving combinations or <u>permutations</u> (e.g., How many different ways can eight students place first, second, and third in a race?) using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or ^{sc} others).
M:DSP:7:5	For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the theoretical probability of an event and tests the prediction through experiments and simulations; and <u>compares and contrasts theoretical and experimental probabilities</u> .
M:DSP:7:5	For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an event in a problem-solving situation.
M:DSP:7:6	In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested while <u>considering the limitations that could affect interpretations</u> ; and when appropriate makes predictions; and asks new questions and makes connections to real world situations. (IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:7:2.</i>)

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills - 6-8
M:PRP:8:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Use problem-solving strategies appropriately and effectively for a given situation. • Determine, collect and organize the relevant information needed to solve real-world problems. • Apply integrated problem-solving strategies to solve problems in the physical, natural, and social sciences and in pure mathematics. • Use technology when appropriate to solve problems. • Reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.
M:PRP:8:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Draw logical conclusions and make generalizations using deductive and inductive reasoning. • Formulate, test, and justify mathematical conjectures and arguments. • Construct and determine the validity of a mathematical argument or a solution. • Apply mathematical reasoning skills in other disciplines

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ 6-8
M:CCR:8:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.
M:CCR:8:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Use models and technology to develop equivalent representations of the same mathematical concept. • Use and create representations to solve problems and organize their thoughts and ideas. • Convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).

Grade Span - 6 – 8

Mathematics

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade
Stem Number

Concepts and Skills - Grades 6-8

M:CCR:8:3

Students will recognize, explore, and develop mathematical connections and be able to:

- Connect new mathematical ideas to those already studied and build upon them.
- Understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon).
- Explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.
- Explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:8:1	Demonstrates conceptual understanding of rational numbers with respect to absolute values, perfect square and cube roots, and percents as a way of describing change (percent increase and decrease) using explanations, models, or other representations.
M(N&O):8:2	Demonstrates understanding of the relative magnitude of numbers by ordering or comparing rational numbers, <u>common irrational numbers</u> (e.g., $\sqrt{2}$, π), numbers with whole number or <u>fractional bases</u> and whole number exponents, <u>square roots</u> , absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality
M:N&O:8:3	No standard at this grade

Grade - 8		Mathematics
Mathematics: <i>Number and Operations</i> continued Grade Level Expectations		
Content Strand Grade Stem Number	Concepts and Skills	
M:N&O:8:4	<p>Accurately solves problems involving proportional reasoning (<u>percent increase or decrease, interest rates, markups, or rates</u>); multiplication or division of integers; and <u>squares, cubes, and taking square or cube roots</u>.</p> <p>(IMPORTANT: <u>Applies the conventions of order of operations.</u>)</p>	
M:N&O:8:5	No standard at this grade	
M:N&O:8:6	<p>Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates benchmark perfect squares and related square roots (e.g., 1^2, 2^2, ... , 12^2, 15^2, 20^2, 25^2, 100^2, 1000^2); determines the part of a number using benchmark percents and related fractions (1%, 10%, 25%, $33\frac{1}{3}\%$, 50%, $66\frac{2}{3}\%$, 75%, and 100%) (e.g., 25% of 16; $33\frac{1}{3}\%$ of 330).</p> <p>(IMPORTANT: <i>The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>	
M:N&O:8:7	<p>Makes estimates in a given situation (including tips, discounts, tax, and the value of a non-perfect square root as between two whole numbers) by identifying when estimation is appropriate, selecting the appropriate method of estimation; determining the level of accuracy needed given the situation; analyzing the effect of the estimation method on the accuracy of results; and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>	
M:N&O:8:8	<p>Applies properties of numbers (odd, even, positive, negative, remainders, divisibility, and <u>prime factorization</u>) and field properties (commutative, associative, identity [<u>including the multiplicative property of one, e.g., $2^0 \times 2^3 = 2^{0+3} = 2^3$, so $2^0 = 1$</u>], distributive, inverses) to solve problems and to simplify computations, and demonstrates conceptual understanding of field properties as they apply to subsets of real numbers <u>when addition and multiplication are not defined in the traditional ways (e.g., if $a \Delta b = a + b - 1$, is Δ a commutative operation?)</u></p>	

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:8:1	Uses properties of angle relationships resulting from two or three intersecting lines (adjacent angles, vertical angles, straight angles, or angle relationships formed by two non-parallel lines cut by a transversal), or two parallel lines cut by a transversal to solve problems.
M:G&M:8:2	<u>Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations.</u>
M:G&M:8:3	No standard at this grade
M:G&M:8:4	No standard at this grade
M:G&M:8:5	<u>Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate.</u>

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:8:6	Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, <u>cylinders</u> , <u>pyramids</u> , or <u>cones</u> . Expresses all measures using appropriate units.
M:G&M:8:7	No standard at this grade
M:G&M:8:8	No standard at this grade
M:G&M:8:9	No standard at this grade
M:G&M:8:10	No standard at this grade

Mathematics: *Functions and Algebra*
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:8:1	Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; and generalizes a linear relationship (non-recursive explicit equation); generalizes a linear relationship to find a specific case; <u>generalizes a nonlinear relationship using words or^{sc} symbols</u> ; or <u>generalizes a common nonlinear relationship to find a specific case</u> .
M:F&A:8:2	Demonstrates conceptual understanding of linear relationships ($y = kx$, $y = mx + b$) as a constant rate of change by solving problems involving the relationship between slope and rate of change; <u>informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations</u> ; or <u>describing the meaning of slope and intercept in context</u> ; and <u>distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations</u> ; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and <u>varying rates of change</u> .
M:F&A:8:3	Demonstrates conceptual understanding of algebraic expressions by evaluating and <u>simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers)</u> ; or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 7\sqrt{x} + 2x$).

Mathematics: *Functions and Algebra* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:8:4	<p>Demonstrates conceptual understanding of equality by showing equivalence between two expressions (<u>expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level</u>) using models or different representations of the expressions, <u>solving formulas for a variable requiring one transformation (e.g., $d = rt$, $d/r = t$)</u>; by <u>solving multi-step linear equations with integer coefficients</u>; by <u>showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution</u>; and by <u>informally solving problems involving systems of linear equations in a context</u>.</p>

Mathematics: *Data, Statistics, and Probability*
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:8:1	<p>Interprets a given representation (line graphs, <u>scatter plots</u>, histograms, or <u>box-and-whisker plots</u>) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:8:2.)</p>
M:DSP:8:2	<p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, <u>quartile values</u>, or <u>estimated line of best fit</u> to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, <u>random</u>, or <u>non-random</u>).</p>
M:DSP:8:3	<p>Organizes and displays data using scatter plots to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems; or identifies representations or elements of representations that best display a given set of data or situation, consistent with the representations required in <u>M:DSP:8:1</u>.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M:DSP:8:2.)</p>
M:DSP:8:4	<p>Uses counting techniques to solve problems in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or^{sc} others).</p>

Grade - 8

Mathematics

Mathematics: *Data, Statistics, and Probability*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:8:5	For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an event in a problem-solving situation; and predicts the theoretical probability of an event and tests the prediction through experiments and simulations; and compares and contrasts theoretical and experimental probabilities; finds the odds of an event and understands the relationship between probability and odds.
M:DSP:8:6	In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested while <u>considering the limitations that could affect interpretations</u> ; and when appropriate makes predictions; and asks new questions and makes connections to real world situations. (IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:8:2.</i>)

Mathematics: *Problem Solving, Reasoning, and Proof*
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills - 6-8
M:PRP:8:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Use problem-solving strategies appropriately and effectively for a given situation. • Determine, collect and organize the relevant information needed to solve real-world problems. • Apply integrated problem-solving strategies to solve problems in the physical, natural, and social sciences and in pure mathematics. • Use technology when appropriate to solve problems. • Reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.
M:PRP:8:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Draw logical conclusions and make generalizations using deductive and inductive reasoning. • Formulate, test, and justify mathematical conjectures and arguments. • Construct and determine the validity of a mathematical argument or a solution. • Apply mathematical reasoning skills in other disciplines

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills Grades ~ 6-8
M:CCR:8:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.
M:CCR:8:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Use models and technology to develop equivalent representations of the same mathematical concept. • Use and create representations to solve problems and organize their thoughts and ideas. • Convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills - Grades 6-8
M:CCR:8:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none"> • Connect new mathematical ideas to those already studied and build upon them. • Understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon). • Explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model. • Explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation(s) to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:HS:1	Demonstrates conceptual understanding of rational numbers by knowing why a real number is rational if and only if the number's decimal expansion eventually repeats or terminates.
M:N&O:HS:2	Demonstrates understanding of the relative magnitude of real numbers by solving problems that involve ordering or comparing elements of any subset of the real numbers.
M:N&O:10:2	Demonstrates understanding of the relative magnitude of real numbers by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., $\sqrt{2}$, π), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.

Mathematics: *Number and Operations*
Grade Level Expectations

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Content Strand Grade Stem Number	Concepts and Skills
M:N&O:HS:1	Demonstrates conceptual understanding of rational numbers by knowing why a real number is rational if and only if the number's decimal expansion eventually repeats or terminates.
M:N&O:HS:2	Demonstrates understanding of the relative magnitude of real numbers by solving problems that involve ordering or comparing elements of any subset of the real numbers.
M:N&O:10:2	Demonstrates understanding of the relative magnitude of real numbers by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., $\sqrt{2}$, π), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:HS:3	No standard at this level
M:N&O:HS:4	<p>Accurately solves problems.</p> <ul style="list-style-type: none"> • Interprets and computes with rational exponents and their relation to radicals, by hand in simple cases (e.g., $4^{3/2}$), and using a calculator when appropriate. • Interprets and computes in scientific notation with and without a calculator. • Solves compound interest problems using $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where n is finite.
M:N&O:10:4	<p>Accurately solves problems involving rational numbers within mathematics, across content strands, disciplines or contexts (with emphasis on, but not limited to, proportions, percents, ratios, and rates).</p>
M:N&O:HS:5	No standard at this level
M:N&O:HS:6	<p>Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers.</p> <p><i>(IMPORTANT: The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)</i></p>

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:HS:7	<p>Makes estimates in a given situation (e.g., tips, discounts, tax, the value of a non-perfect square root or cube root) by identifying when estimation is appropriate, selecting the appropriate method of estimation; determining the level of accuracy needed given the situation; analyzing the effect of the estimation method on the accuracy of results; evaluating the reasonableness of solutions appropriate to GSEs across content strands.</p> <p><i>(IMPORTANT: The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.)</i></p>
M:N&O:HS:8	<p>Applies properties of numbers and field properties (including determining whether a given subset of numbers is closed under a given arithmetic operation) to solve problems or to simplify computations; and compares and contrasts the properties of numbers and number systems; adds and multiplies numerical matrices with attention to the arithmetic properties of these operations.</p>

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:HS:1	No standard at this level
M:G&M:HS:2	<p>Creates formal proofs of propositions (e.g., angles, lines, circles, distance, midpoint and polygons including triangle congruence and similarity).</p> <p><i>(IMPORTANT: It is the intent that students are creating formal proofs as articulated in the process standards, independent of the topic being studied. Furthermore, students should not be limited to any particular method of proof, but rather use a variety of strategies and those that work best for them. Some topics may be treated more formally than others.)</i></p>
M:G&M:10:2	<div style="border: 2px solid black; padding: 10px;"> <p>Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem).</p> </div>
M:G&M:HS.3	No standard at this level

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:HS:4	Applies the concepts of congruency by using matrices to represent reflections, translations, and rotations.
M:G&M:10:4	Applies the concepts of congruency by solving problems on or off a coordinate plane involving reflections, translations, or rotations; or solves problems using congruency involving problems within mathematics or across disciplines or contexts.
M:G&M:HS:5	Applies concepts of similarity to define the trigonometric functions as ratios of sides of right triangles; uses the ratios of the sides of special right triangles ($30^\circ - 60^\circ - 90^\circ$ and $45^\circ - 45^\circ - 90^\circ$) to determine the sine, cosine and tangent of 30° , 45° , and 60° ; and solves related problems.
M:G&M:10:5	Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts.
M:G&M:HS:6	Applies trigonometric formulas (e.g., Law of Sines, Law of Cosines, $A = \frac{1}{2} ab \sin C$) to find angles, lengths and areas of polygons.
M:G&M:10:6	Solves problems involving perimeter, circumference, or area of two-dimensional figures (including composite figures) or surface area or volume of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:HS:7	Applies informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations (e.g., use successive approximation to find the area of a pond); and uses measurement conversion strategies (e.g., unit/dimensional analysis).
M:G&M:10:7	Uses units of measure appropriately and consistently when solving problems across content strands; makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations involving measurement in other GSEs.
M:G&M:HS:8	No standard at this level
M:G&M:HS:9	Solves problems on and off the coordinate plane involving distance, midpoint, perpendicular and parallel lines, or slope.
M:G&M:HS:10	Demonstrates conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three-dimensional objects, and by solving related problems; perform and justify constructions with a compass and straightedge or dynamic geometric software.

Mathematics: Functions and Algebra
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:HS:1	Identifies arithmetic and geometric sequences and finds the n th term; then uses the generalization to find a specific term.
M:F&A:10:1	<div style="border: 2px solid black; padding: 5px;"> Identifies, extends, and generalizes a variety of patterns (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations. </div>
M:F&A:HS:2	<p>Demonstrates conceptual understanding of linear and nonlinear functions and relations.</p> <ul style="list-style-type: none"> • Analyzes characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change. • Understands one-to-one (injective) functions and that a function that is one-to-one has a converse that is also a function; and finds inverses algebraically and graphically. • Graphs polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes. • Applies knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in the natural and social sciences.

Mathematics: Functions and Algebra continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:10:2	<p>Demonstrates conceptual understanding of linear and nonlinear functions and relations (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).</p>
M:F&A:HS:3	<p>Demonstrates conceptual understanding of algebraic expressions.</p> <ul style="list-style-type: none"> • Manipulates, evaluates, and simplifies algebraic and numerical expressions. • Adds, subtracts, multiplies and divides polynomials and rational expressions. • Factors quadratic and higher degree polynomials. • Understands properties of logarithms and converts between logarithmic and exponential forms. • Manipulates, evaluates, and simplifies expressions involving rational exponents and radicals and converts between expressions with rational exponents and expressions with radicals. • Understands the effect of simplifying rational expressions on the domain of the related functions (e.g., $x^2/x = x$ for $x \neq 0$).
M:F&A:10:3	<p>Demonstrates conceptual understanding of algebraic expressions by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions.</p>

Mathematics: *Functions and Algebra* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:HS:4	<p>Demonstrates conceptual understanding of equality.</p> <ul style="list-style-type: none"> • Factors, completes the square, uses the quadratic formula, and graphs quadratic functions to solve quadratic equations. • Solves equations involving polynomial, rational, and radical expressions. Graphs and interprets the solutions. • Understands extraneous solutions. • Finds approximate solutions to equations by graphing each side as a function using technology. Understands that any equation in x can be interpreted as the equation $f(x) = g(x)$ and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$. • Solves 2×2 and 3×3 systems of linear equations and graphically interprets the solutions. • Solves systems of linear and quadratic inequalities. • Solves systems of equations involving nonlinear expressions and graphically interprets the solutions. • Translates problem situations into inequalities; and solves linear and non-linear inequalities (symbolically and graphically).
M:F&A:10:4	<p>Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.</p>

Grade Span – High School		Mathematics
Mathematics: <i>Data, Statistics, and Probability</i> Grade Level Expectations		
<p>Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.</p>		
Content Strand Grade Stem Number	Concepts and Skills	
M:DSP:HS:1	<p>Interprets a given representation(s) (e.g., regression function including linear, quadratic, and exponential) to analyze the data to make inferences and to formulate, justify, and critique conclusions.</p> <p>(IMPORTANT: <i>Analyze data consistent with concepts and skills in M:DSP:HS:2</i>).</p>	
M:DSP:10:1	<p>Interprets a given representation(s) (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g., media, workplace, social and environmental situations).</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:10:2</i>).</p>	
M:DSP:HS:2	<p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of dispersion (standard deviation, variance, and percentiles).</p>	

Mathematics: *Data, Statistics, and Probability* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:10:2	<p>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed.</p>
M:DSP:HS:3	<p>Organizes and displays one- and two-variable data using a variety of representations (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts, linear, quadratic, and exponential regression functions) to analyze the data to formulate or justify conclusions, make predictions, or to solve problems with or without using technology.</p>
M:DSP:10:3	<p>Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in M:DSP:10:1.</p>
M:DSP:HS:4	<p>Uses counting techniques to solve problems in context involving combination or permutations using a variety of strategies (e.g., nCr, nPr, or $n!$); and finds unions, intersections, and complements of sets.</p>
M:DSP:10:4	<p>Uses counting techniques to solve problems in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or^{sc} others).</p>

Grade Span – High School		Mathematics
Mathematics: <i>Data, Statistics, and Probability</i> continued Grade Level Expectations		
Content Strand Grade Stem Number	Concepts and Skills	
M:DSP:HS:5	For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the theoretical probability of an event and tests the prediction through experiments and simulations; compares and contrasts theoretical and experimental probabilities; finds the odds of an event and understands the relationship between probability and odds.	
M:DSP:10:5	Solves problems involving experimental or theoretical probability.	
M:DSP:HS:6	<p>In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, research, experimentation) and sampling techniques (e.g., random sample, stratified random sample) to collect the data necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the questions or hypotheses being tested while considering the limitations of the data that could effect interpretations; and when appropriate makes predications, asks new questions, or makes connections to real-world situations.</p> <p>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M:DSP:10:2.</i>)</p>	

Mathematics: Problem Solving, Reasoning, and Proof
Grade Level Expectations

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum (as well as other areas of study) and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world using data that are meaningful to them and performing mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Content Strand Grade Stem Number	Concepts and Skills ~ High School
M:PRP:HS:1	<p>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:</p> <ul style="list-style-type: none"> • Expand the repertoire of problem-solving strategies and use those strategies in more sophisticated ways. • Use technology whenever appropriate to solve real-world problems (e.g., personal finance, wages, banking and credit, home improvement problems, measurement, taxes, business situations, purchasing, and transportation). • Formulate and redefine problem situations as needed to arrive at appropriate conclusions.
M:PRP:HS:2	<p>Students will use mathematical reasoning and proof and be able to:</p> <ul style="list-style-type: none"> • Expand the repertoire of proof techniques and use those techniques in more sophisticated ways. • Use informal and formal reasoning and proof to explain and justify conclusions. • Formalize mathematical arguments through the use of deductive reasoning. • Use the principle of mathematical induction. • Use reasoning and proof throughout classroom discussions independent of the mathematical topic being studied. • Recognize how reasoning and proof influence the structure of mathematics.

Mathematics: *Communication, Connections, and Representations*
Grade Level Expectations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

Content Strand Grade Stem Number	Concepts and Skills ~ High School
M:CCR:HS:1	<p>Students will communicate their understanding of mathematics and be able to:</p> <ul style="list-style-type: none"> • Explain and justify their thinking and develop increasingly sophisticated questions for given problem-situations. • Critique and follow the logic of arguments presented within mathematics and across disciplines.
M:CCR:HS:2	<p>Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:</p> <ul style="list-style-type: none"> • Choose appropriate representations and mathematical language (e.g., spreadsheets, geometric models, algebraic symbols, tables, graphs, matrices) to present ideas clearly and logically for a given situation. • See a common structure in mathematical phenomena that come from very different contexts (e.g., the sum of the first n odd natural numbers, the areas of square gardens, and the distance traveled by a vehicle that starts at rest and accelerates at a constant rate can be represented by functions of the form $f(x) = ax^2$). • Find representations that model essential features of a mathematical situation (e.g., cost of postage can be modeled by a step-function). • Use representations as a primary means for expressing and understanding more abstract mathematical concepts.

Grade Span – High School

Mathematics

Mathematics: *Communication, Connections, and Representations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills ~ High School
M:CCR:HS:3	<p>Students will recognize, explore, and develop mathematical connections and be able to:</p> <ul style="list-style-type: none">• Explain in oral or written form how mathematics connects to other disciplines, to daily life, careers, and society (e.g., geometry in art and literature, data analysis in social studies, and exponential growth in finance).• Explain multiple approaches that lead to equivalent results when solving problems.

Mathematics: *Number and Operations*
Grade Level Expectations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation(s) to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:AM:1	Demonstrates conceptual understanding of the real number system as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number given in fractional form eventually repeats or terminates (without using a calculator).
M:N&O:AM:2	No standard at this level
M:N&O:HS:3	No standard at this level

Mathematics: *Number and Operations* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:N&O:AM:4	<p>Accurately solves problems.</p> <ul style="list-style-type: none"> • Solves problems involving scientific notation and assesses the precision of the answer in terms of significant digits. • Solves problems involving complex numbers by adding, multiplying, dividing, and computing conjugates; interprets complex numbers geometrically; and understands complex numbers as an extension of the real numbers (e.g., arising in solutions of polynomial equations). • Manipulates complex numbers using rectangular and polar coordinates. • Finds the nth roots of a complex number using De Moivre's Theorem. • Solves compound interest problems including continuously compounded interest
M:N&O:AM:5	No standard at this level
M:N&O:AM:6	No standard at this level
M:N&O:AM:7	No standard at this level
M:N&O:AM:8	Explores field properties with vectors and matrices; algebraically/geometrically interpret vectors, vector addition, and scalar multiplication in the plane; and solves problems using matrices (e.g., inverses, identity, determinants, and augmented matrices).

Mathematics: *Geometry and Measurements*
Grade Level Expectations

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:AM:1	No standard at this level
M:G&M:AM:2	Extends and deepens knowledge and usage of proofs and proof techniques; and uses geometric models to represent and distinguish between Euclidean and Non-Euclidean Systems.
M:G&M:AM:3	No standard at this level
M:G&M:AM:4	No standard at this level
M:G&M:AM:5	No standard at this level
M:G&M:AM:6	Derives and uses formulas for lengths of arcs and areas of sectors and areas of segments of circles.

Mathematics: *Geometry and Measurements* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:G&M:AM:7	Uses radian measure appropriately when solving problems; converts between radian measure and degree measure; and understands why radian measure is useful.
M:G&M:AM:8	No standard at this level
M:G&M:AM:9	Solves problems using analytic geometry (including three-dimensions) and circular trigonometry (e.g., find the equation of a circle inscribed in a triangle; find the distance between opposite vertices in a rectangular solid); explores and interprets the characteristics of conic sections graphically and algebraically including understanding how different planar slices of a double cone yield different conic sections; knows the characterization of conic sections as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for the conic sections.
M:G&M:AM:10	No standard at this level

Mathematics: *Functions and Algebra*
Grade Level Expectations

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:AM:1	Computes partial sums of infinite arithmetic and geometric sequences, determines when an infinite geometric series converges, and finds its sum. Connects arithmetic and geometric sequences to linear and exponential functions, respectively. Works between recursive and explicit representations.
M:F&A:AM:2	<p>Demonstrates conceptual understanding of linear and nonlinear functions and relations.</p> <ul style="list-style-type: none"> • Understands functions and relations from a set-theoretic perspective, and operations on functions including composition. • Analyzes characteristics of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes; graphs classes of functions; and understands domain restrictions and their effects on functions. • Analyzes properties of functions including onto (surjectivity), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither. • Analyzes informally the idea of continuity and limits.

Grade – High School

Advanced Mathematics

Mathematics: *Functions and Algebra* continued
Grade Level Expectations

Content Strand Grade Stem Number	Concepts and Skills
M:F&A:AM:3	<p>Demonstrates conceptual understanding of algebraic expressions.</p> <ul style="list-style-type: none">• Simplifies complex fractions.• Uses the Remainder Theorem, the Factor Theorem and Rational Root Theorem for polynomials.• Knows the Fundamental Theorem of Algebra and that non-constant polynomials always factor into linear factors over the complex numbers.• Understands the difference between factoring polynomials over integer, rational, real, and complex numbers.
M:F&A:AM:4	<p>Demonstrates conceptual understanding of equality.</p> <ul style="list-style-type: none">• Solves equations and verifies/proves identities involving trigonometric expressions.• Solves equations involving exponential and logarithmic expressions; graphs and interprets the solutions.• Uses matrices or determinants to solve systems of equations with or without technology.• Knows and applies the Intermediate Value Theorem to find exact or approximate solutions of equations or zeros of continuous functions.

Mathematics: *Data, Statistics, and Probability*
Grade Level Expectations

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Content Strand Grade Stem Number	Concepts and Skills
M:DSP:AM:1	No standard at this level
M:DSP:AM:2	Analyzes and interprets measures of dispersion (standard deviation, variance, and percentiles) and central tendency for the normal distribution; and interprets the correlation coefficient and the coefficient of determination in the context of data.
M:DSP:AM:3	Uses technology to explore the method of least squares and median-median for linear regression.
M:DSP:AM:4	No standard at this level
M:DSP:AM:5	Solves probability problems (e.g., by applying concepts of counting, random variables, independence/dependence of events, and conditional probability).
M:DSP:AM:6	No standard at this level